

# 4.7

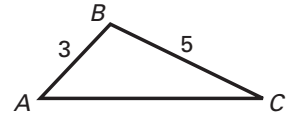
## Triangle Inequalities

**Goal** Use triangle measurements to decide which side is longest and which angle is largest.

### THEOREM 4.10

**Words** If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

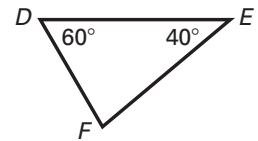
**Symbols** If  $BC > AB$ , then  $m\angle A > m\angle C$ .



### THEOREM 4.11

**Words** If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

**Symbols** If  $m\angle D > m\angle E$ , then  $EF > DF$ .



### Example 1 Order Angle Measures

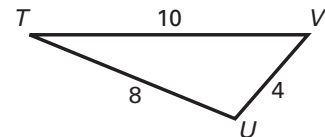
Name the angles from largest to smallest.

#### Solution

$TV > TU$ , so  $m\angle U > m\angle V$ .

$TU > UV$ , so  $m\angle V > m\angle T$ .

**Answer** The order of the angles from largest to smallest is  $\angle U$ ,  $\angle V$ ,  $\angle T$ .



**Example 2**    **Order Side Lengths**

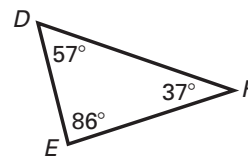
Name the sides from longest to shortest.

**Solution**

$$m\angle E > m\angle D, \text{ so } \overline{DF} > \overline{FE}.$$

$$m\angle D > m\angle F, \text{ so } \overline{FE} > \overline{DE}.$$

**Answer** The order of the sides from longest to shortest is  $\overline{DF}$ ,  $\overline{FE}$ ,  $\overline{DE}$ .

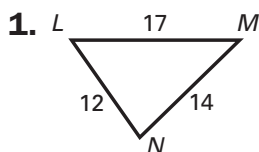
**Follow-Up**

Which theorem is used in Example 1?

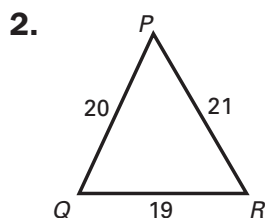
**Theorem 4.10**

Which theorem is used in Example 2?

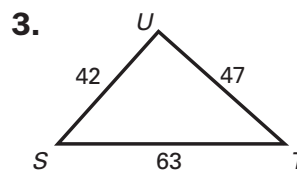
**Theorem 4.11**

**✓ Checkpoint** Name the angles from largest to smallest.

$\angle N, \angle L, \angle M$

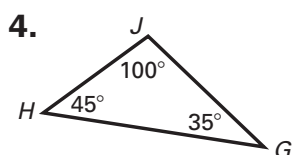


$\angle Q, \angle R, \angle P$

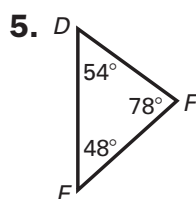


$\angle U, \angle S, \angle T$

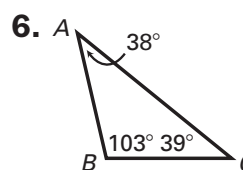
Name the sides from longest to shortest.



$\overline{HG}, \overline{JG}, \overline{HJ}$



$\overline{DE}, \overline{EF}, \overline{DF}$



$\overline{AC}, \overline{AB}, \overline{BC}$

### THEOREM 4.12: TRIANGLE INEQUALITY

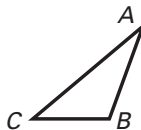
**Words** The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

**Symbols**

$$CA + AB > BC$$

$$AB + BC > CA$$

$$BC + CA > AB$$



#### Example 3 Use the Triangle Inequality

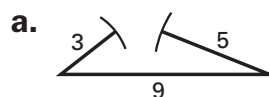
Can the side lengths form a triangle? Explain.

a. 3, 5, 9

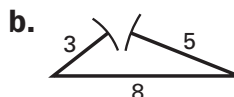
b. 3, 5, 8

c. 3, 5, 7

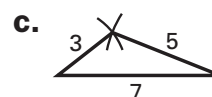
**Solution**



These lengths do not form a triangle, because  $3 + 5 < 9$ .



These lengths do not form a triangle, because  $3 + 5 = 8$ .



These lengths do form a triangle, because  $3 + 5 > 7$ ,  $3 + 7 > 5$ , and  $5 + 7 > 3$ .

✓ **Checkpoint** Can the side lengths form a triangle? Explain.

7. 5, 7, 13

No;  
 $5 + 7 < 13$

8. 6, 9, 12

Yes;  
 $6 + 9 > 12$   
 $6 + 12 > 9$   
 $9 + 12 > 6$

9. 10, 15, 25

No;  
 $10 + 15 = 25$